

JETS, FLOW, GAPS: NON-GLOBAL EFFECTS

Giuseppe Marchesini

Dipartimento di Fisica, Università di Milano-Bicocca and
INFN, Sezione di Milano, Italy

I discuss non-global observables in jet physics which contain single logarithmic contributions of non standard type (non-global logs).

Perturbation theory (PT) provides the most useful method to study QCD [1]. Although not sufficient to provide a full understanding of QCD, we still miss the comprehension for hadronization, non-perturbative corrections and non convergence of PT expansion, PT study of the general structure of QCD radiation provides most valuable information. Typical example is the study of jet shape observables in e^+e^- such as thrust, broadening, C -parameter, jet mass, out-of-event plane momentum. These distributions are computed at high PT accuracy: two-loop exact calculations and resummation of double and single logarithms leading to Sudakov form factors. By comparing with experiments one can argue about features of NP corrections. This strategy has been very successful especially in e^+e^- .

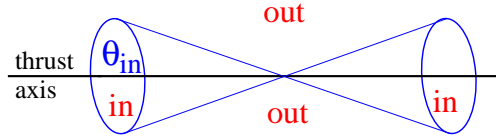
The accuracy reached in e^+e^- cannot be easily exported to DIS or hadronic collider reactions. The reason is that, contrary to e^+e^- , here jet-shape observables are in general *non-global*, i.e. one needs to limit the phase space region where to register the hadronic radiation. It was a new discovery by Dsgupta and Salam [2] that for these observables there are single logarithmic enhanced pieces, *non-global logs*, which are beyond the ones we know how to resum in the study of standard jet shape (*global*) observables (defined in the full phase space). As we will discuss, *non-global logs* are due to correlated soft emission at large angle. They have a pure soft origin (no collinear singularities are present) and then can be studied by using the multi soft gluon emission distribution known in the large N_C limit [3]. Non-global logs can be resummed by a non-linear evolution equation [4] for which the solution is only numerical or very asymptotic.

Non-global logs are present for instance in Stermann-Weiberg [5] distributions (energy in a cones), “isolated” photon distributions, inter jet string/drag effects, profile of a separate jet (e.g. current hemisphere). In general they enter all DIS and hadronic collision distributions in which one needs rapidity cuts.

To illustrate the QCD dynamics of non-global effects consider the following non-global distribution in e^+e^-

$$\Sigma_{e^+e^-}(E_{\text{out}}) = \sum_n \int \frac{d\sigma_n}{\sigma_T} \Theta \left(E_{\text{out}} - \sum_{h \in \text{out}} p_{th} \right), \quad (1)$$

with the sum restricted to hadrons registered in the region “out” away from the thrust axis



Typically one has $E_{\text{out}} \ll Q$ so that powers of $\ln Q/E_{\text{out}}$ need to be resummed. Since the phase space is divided into two regions we need to consider two cases: the soft gluon which is emitted off the $q\bar{q}$ pair does or does not enter the “out” region. In the first case, due to real-virtual cancellation, successive gluon branching can be neglected. This is the bremsstrahlung component, present also in global distributions giving Sudakov form factors.

In the second case the gluon needs to branch in order to enter the registered region. Branching $g \rightarrow gg$ duplicates gluons and then this component is described by a non-linear equations which can be formulated in the large N_C limit as we will discuss. Clearly this component is absent in the case of global observables (soft gluon emitted off the $q\bar{q}$ pair enters always the detected region). The two components give

$$\Sigma_{e^+e^-}(E_{\text{out}}) = S_{q\bar{q}}(\Delta) \cdot C_{q\bar{q}}(\Delta), \quad \Delta = \int_{E_{\text{out}}}^Q \frac{dk_t}{k_t} \frac{N_c \alpha_s(k_t)}{\pi}, \quad (2)$$

with $S_{q\bar{q}}(\Delta)$ the (single log) Sudakov factor and $C_{q\bar{q}}(\Delta)$ the secondary branching contribution.

The evolution equation involving both components is obtained from the

multi soft gluon distributions [3]

$$e^+e^- \rightarrow p\bar{p} k_1 k_2 \dots k_n, \quad d\sigma_n \sim \frac{(p\bar{p})}{(pk_1)(k_1 k_2) \dots (k_n \bar{p})}, \quad (3)$$

with $p\bar{p}$ the primary $q\bar{q}$ dipole emitting soft gluons k_i (sum over permutations is understood). Introducing the general distribution Σ_{ab} for the emission off a hard dipole ab with moment $p_a p_b$ inside the jet region “in”, from (3) one deduces [4]

$$\partial_\Delta \Sigma_{ab} = -\partial_\Delta R_{ab} \cdot \Sigma_{ab} + \int_{\text{in}} dN_{ab \rightarrow k} \left(\Sigma_{ak} \Sigma_{kb} - \Sigma_{ab} \right), \quad (4)$$

where

$$R_{ab} = \int_{E_{\text{out}}}^E \frac{d\omega}{\omega} \frac{N_c \alpha_s(\omega)}{\pi} \int_{\text{out}} dN_{ab \rightarrow k}, \quad dN_{ab \rightarrow k} \equiv \frac{(1 - \cos \theta_{ab}) d\Omega_k}{4\pi(1 - \cos \theta_{ak})(1 - \cos \theta_{kb})}. \quad (5)$$

The physical e^+e^- distribution is obtained by replacing $p_a p_b$ with the primary dipole $p\bar{p}$. The first term R_{ab} is the “radiator” (single soft emission inside the register region “out”) giving the Sudakov component $S_{ab} = e^{-R_{ab}}$, see (2).

The integral term in (4) corresponds to branching: the dipole ab splits into the two dipoles ak and kb generated by the emission of a soft gluon k which must stay inside the not registered region “in” (otherwise real-virtual cancellations take place). Each new dipole ak or kb subsequently radiates and leads to Σ_{ak} or Σ_{kb} . When the soft gluon emerges into the registered region “out” there is no further branching. Virtual contributions enter the second term in the integrand. Collinear singularities in the dipole splitting function $dN_{ab \rightarrow k}$ for k parallel to p_a cancels since $\Sigma_{ak} \rightarrow 1$ and $\Sigma_{kb} \rightarrow \Sigma_{ab}$. Similarly for k parallel to p_b .

It is clear from (4) that one needs to study Σ_{ab} in both cases with $p_a p_b$ in the opposite and in the same hemisphere. Actually, the behaviour for large Δ (a remote experimental possibility) is determined from the peak of Σ_{ab} for $\theta_{ab} \rightarrow 0$. Asymptotically one has the scaling behaviour

$$\Sigma_{ab}(\Delta) \simeq h(z), \quad z = \frac{\theta_{ab}^2}{2\theta_{\text{crit}}^2(\Delta)}, \quad \theta_{\text{crit}} \sim e^{-c\Delta}, \quad (6)$$

with c a determinable constant. The result is that the branching takes place with very little dispersion away from the direction of the primary emitting parton. There is than a large buffer [2].

It has been observed by Al Mueller [6] that (4) has a structure surprisingly similar to the Kovchegov equation [7] for the S -matrix at high energy. This should not be an accident since also the Kovchegov equation is based on gluon multiplication (although in the exchanged channel). Their connections will be further studied¹.

Non-global logs originate from branching within the region “in” close to the jets and therefore, if one is able to inhibit emission within this region, they do not appear. Using this fact Berger, Kúcs and Sterman [8] introduced a *flow-shape* correlation in which, together with E_{out} one considers a global variable, for instance $\tau = 1 - T$ with T the thrust. To inhibit radiation within the jet region one takes $\tau \ll 1$. Considers the following three distributions

$$\begin{aligned} \tau - \sum_i \tau(k_i), \quad E_{\text{out}} - \sum_{i \in \text{out}} k_{ti} &\Rightarrow \Sigma_{fs}(Q, \tau, E_{\text{out}}) && \text{flow-shape} \\ \tau - \sum_i \tau(k_i), &\Rightarrow \Sigma(Q, \tau) && \text{global} \\ E_{\text{out}} - \sum_{i \in \text{out}} k_{ti} &\Rightarrow \Sigma_{\text{out}}(Q, E_{\text{out}}) && \text{non-global} \end{aligned}$$

with the last being (1). For $\tau \simeq E_{\text{out}}/Q \ll 1$ they found $\Sigma_{fs}(Q, \tau, E_{\text{out}}) \simeq \Sigma(Q, \tau)$ as expected. Corrections are given by power of $\ln E_{\text{out}}/\tau$. Actually it has been shown [9] that these logs can be resummed and one ends up with a nice factorized formula valid for small τ and E_{out}/Q without further restrictions

$$\Sigma_{fs}(Q, \tau, E_{\text{out}}) = \Sigma(Q, \tau) \cdot \Sigma_{\text{out}}(\tau Q, E_{\text{out}}). \quad (7)$$

This result is obtained by analysing the effective kinematical restriction of the multi gluon emission (after cancellations of virtual corrections) are taken

¹Note added. Kovchegov equation for the S -matrix and equation (4) for Σ_{ab} with small θ_{ab} are formally the same. The physics difference is that the first is for a function of transverse coordinates while the second for a function of (small) branching angles. Such a difference is easily explained by the different dominating kinematical configurations for the two problems. The linear regime of Kovchegov equation (away from saturation) is the BFKL equation. On the contrary there is no linear regime for Σ_{ab} . Moving from this observation, it has been introduced [11] a jet physics observable (heavy quark pair production at large angle) dominated by non-global logs which are resummed by BFKL equation (for a function of angles). The distribution is then given in terms of hard Pomeron intercept. It is a big surprise to see that observables in jet physics involve BFKL dynamics, although in angles instead of transverse coordinates.

into account. One finds that the allowed kinematical region for the flow-jet distribution is given by the sum of the one for the global distribution in τ at the same hard scale Q plus the one for the non-global distribution in E_{out} , but with reduced hard scale at τQ .

In some cases, non-global logs (in non-global observables) can be avoided. I give a couple of examples [10] in DIS in which non-global logs can be avoided although rapidity cuts are present. We consider dijet events in DIS with large P_t (in the Breit frame). The first observable is the azimuthal correlation

$$H(\chi) = \sum_{hh'} \frac{p_{th} p_{th'}}{Q^2} \delta(\chi - \chi_{hh'}), \quad (8)$$

with $\chi_{hh'}$ back-to-back azimuth angle. It is similar to EEC in e^+e^- in which, due to the different geometry, one measures polar angles instead of azimuthal ones. Although the sum is over the full phase space, the relevant contributions to $H(\chi)$ come from non soft hadrons emitted with rapidities close to the dijet. Therefore, a limitation in rapidity regions away from dijet does not call for non-global logs. The standard QCD resummation of global logs can be performed at the highest accuracy leading to the product of three Sudakov factors (one for the incoming and two from the dijet).

The second example is the out-of-plane radiation

$$K_{\text{out}} = \sum_h |p_h^{\text{out}}|, \quad \eta_h > \eta_0, \quad (9)$$

with p_h^{out} the momentum component in the direction orthogonal to the dijet event plane. For experimental reasons the sum is restricted to the rapidity region defined by η_0 . This is clearly a non global observable. However, for large η_0 , the non-global logs contribution is negligible. To estimate how large η_0 should be, observe that contribution to K_{out} from a hadron with rapidity larger than η_0 is (quite) smaller than $Q e^{-\eta_0}$. Therefore, for $K_{\text{out}} > Q e^{-\eta_0}$ one can neglect the rapidity cut in (9) and perform the QCD resummation in terms of Sudakov form factors.

References

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